Part 2: "Surfer Speed vs. Wave Speed and Peel Angle"

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Part 1 was a tutorial on the Physics of Motion and the relationship between Work and Energy. Therefore, Part 1 was written to lay the groundwork for understanding Part 2. Maybe boring for some readers, mind-numbing for others, but necessary for most people, I think. If you digested the material covered in Part 1, then you are ready for Part 2. Thanks for your patience!

Part 2 is where ‘the rubber hits the road’, and we find answers to questions about “Maximum Surfer Speed on a Wave.”

Part 1 also tried to answer the question: “How fast IS the surfer going on a wave?”
Part 2 will try to answer the question: “How fast CAN a surfer go on a wave?”

A surfer on a wave is constantly managing his Energy budget, trading the Potential Energy that he obtained by going up high on the wave, for more Kinetic Energy and therefore higher speed when he needs it, down lower on the wave.

A roller coaster does exactly the same thing. The cars and riders are mechanically lifted to the top of the first, and highest, rise of the tracks. The amount of Potential Energy obtained at the top is exactly identical to the amount of Work that was performed in lifting the combined Weight of the cars and riders up to that Height above their starting point at ground level.

Then, the cars are released and are free to roll downhill, gaining Speed and Kinetic Energy all the way down to the bottom. If there were no drag or friction, the Kinetic Energy at the bottom would be identical to the Potential Energy they started out with at the top. KE = PE.

Because of the total drag (due to friction and air drag) though, their total Energy budget is slowly eroding, and so each succeeding hill is less high, until finally, the cars are brought in to the loading station and brought to a stop with braking action.

When you are sitting in the surf line-up, waiting for the next set, you are starting out in a position, (on average), at Sea Level, where your Potential Energy and Kinetic Energy are both zero.

Then, as the wave you want approaches, it starts lifting you and your board up towards the top of the wave. If it lifts you all the way up to the top of wave, just as you are about to catch it by paddling fast enough for you to start sliding downhill and then match the wave speed, you have now gained Energy of Position, that is, “Potential Energy”, AND, by matching the wave’s speed, you have now ALSO gained Energy of Motion, that is, “Kinetic Energy”. This combination of PE and KE is your Total Energy Budget.

If you don’t paddle for the wave, then the only Energy you have at the top of the wave is PE. But if you DO paddle for the wave, then when you match the wave speed and can stand up, you have gained the energy of motion due to the Wave Speed. I’ll call that “KEwave”. Just remember: that’s YOUR Kinetic Energy when moving at the Wave Speed.

So, at the top of the wave, just as you match the wave speed, but just BEFORE you make the drop, we can say that:

Total Energy, or KEtotal = PE + KEwave

Remember: PE is the WORK that the wave performs by lifting you and your board up to the top of the wave, that is, PE = WORK = Weight times Height = W*H,
where W is the total Weight lifted, and H is the Height lifted to, i.e., the Wave Height.

When you make the drop, but just before you turn, you trade your Potential Energy at the top of the wave for additional Kinetic Energy at the bottom of your drop. At that point, you are moving faster then the Wave Speed. If you could make it all the way to the Bottom of the wave before you turn, you would have KE = PE. Let's call that Kinetic Energy obtained from making the drop, "KEdrop."

Now, our Total Energy, KEtotal = KEwave + KEdrop

I'm going to relabel the Total Energy as the "Surfer Energy", since that's his Total Energy Budget, and is what gives him the ability to go fast across the wave, or race fast sections, or just climb back up high and bang off the lip, or be set up for the next maneuver.

So, now we have: KEsurfer = KEwave + KEdrop

The formula for Kinetic Energy is (1/2)*(W/g)*V^2, or KE = (W/2g)*V^2

Note that KE varies with the SQUARE of the Velocity of the moving body.

So,
For a surfer moving at full trim speed across a wave, KEsurfer = (W/2g)*(Vsurfer)^2

For a surfer at the top of a wave, moving at Wave Speed, KEwave = (W/2g)*(Vwave)^2

For a surfer who just dropped in, about to make his turn, KEdrop = PE = W*H

So, if KEsurfer = KEwave + KEdrop,

Then, (W/2g)*(Vsurfer)^2 = (W/2g)*(Vwave)^2 + W*H

Solving for (Vsurfer)^2, we get:

(Vsurfer)^2 = [(W/2g)*(Vwave)^2 + W*H] / (W/2g)
Or,
(Vsurfer)^2 = [(W/2g)*(Vwave)^2 + W*H] * (2g)/W

Cancelling the W and the 2g terms, we get:

(Vsurfer)^2 = [(Vwave)^2 + 2gH]

Remember that, in a right triangle, c squared = a squared + b squared,
Or, c^2 = a^2 + b^2

So,
In the above result, it looks like we have the SQUARE of the hypotenuse of a right triangle, represented by the SQUARE of the surfer's speed "Vsurfer", with the SQUARE of one side of the triangle represented by the SQUARE of the Wave Speed, "Vwave"; and the SQUARE of the other side of the triangle represented by the SQUARE of the speed of the curl as measured at right angles to the wave speed, given by its equivalent squared value, "2gH".

So, we see that (Vcurl)^2 = 2gH

Now, let's rewrite this solution for Surfer Speed, using the formula for Wave Speed that was given in Part 1 of this document, where:

Vwave = SQRT (g*d),
Or,
Vwave = (g*d)^0.5

Then, (Vwave)^2 = g*d,

But, if we use a Breaker Depth Index value of 1.28, then: d / Hb = 1.28,
So, d = 1.28 times Hb, or: d = 1.28*Hb where "Hb" is the Breaking Wave Height.

Now, we have: (Vwave)^2 = 1.28*g*Hb

Substituting these in the Surfer Speed formula, we know have:

(Vsurfer)^2 = (Vwave)^2 + (Vcurl)^2

So,,
(Vsurfer)^2 = 1.28*g*Hb + 2*g*Hb
(Vsurfer)^2 = (1.28+2)*(g*Hb)
(Vsurfer)^2 = 3.28*(g*Hb)

So, now we have the formula for Maximum Surfer Speed:
Vsurfer = SQRT(3.28*g*Hb)

But look! We can compare this Maximum Surfer Speed to the Wave Speed:

The ratio of (Vsurfer)^2 / (Vwave)^2 = (3.28*g*Hb) / (1.28*g*Hb)
= (3.28/1.28)
= 2.5625

So, Vsurfer / Vwave = SQRT (2.5625) = 1.600781059

It looks like the fastest you could go on any wave is about 1.6 times as fast as the Wave Speed!

Remember that in Part 1 we found that Cos B = (Vwave / Vsurfer)?
That is just the reciprocal of the above ratio of Surfer Speed to Wave Speed.
So,
Cos B = SQRT (1.28/3.28) = SQRT(0.390243902) = 0.624695048

Then,
The Break Angle B is the ANGLE whose Cosine = 0.624695048),

Or,
B = ArcCosine (0.624695048)
B = 51.34019175 Degrees

This is for a surfer cruising in a straight line across a wave at his highest trim speed. If he stays up high on the wave where the highest speed is possible, he would be able to dive lower to gain some additional speed going through a short fast section up ahead, and maybe be able to make it out the other side.

Generalizing in regards to the Breaker Depth Index, BDI, it would appear that the highest surfer speed relative to wave speed is proportional to the square root of the ratio
(BDI +2) / BDI)

So,
Max Surfer Speed / Wave Speed = SQRT[(BDI +2) / BDI]
And,
Maximum Makeable Break or Peel Angle = ArcCosine [BDI / (BDI+2)]

So far, we've only determined the RATIOS of speeds involved in surfing a wave. Let's see what kind of ACTUAL SPEEDS are therefore possible on waves of given heights.

For Makaha Point Break, where the 25 ft waves peel off pretty fast on WNW swells, the Acceleration of Gravity at the Latitude of the Point Line-up is about 32.110407 ft/sec^2.

I'll use the rounded off value of 32.11 f/s^2, because I got the Latitude from Google Earth aerial photos and had to interpolate to estimate the position of the reef block where the line-up is located. I am very familiar with that reef, because I snorkelled out there often for 15 years. It should be accurate to within 10 ft or 3 meters.

If the water depth where 25 ft waves break is 1.28 times the breaking wave height, Hb, then the depth d = 1.28*25 = 32 ft.

Then, the Wave Speed in 32 ft of water, Vwave = SQRT(g*d) = SQRT(32.11* 32)
So,
Vwave,fps = 32.05495282 fps
And,
Vwave,mph = (15/22)*Vwave,fps = 21.85564965 MPH

If Maximum Makeable Break Angle = 51.34019175 degrees,
Then, Maximum Surfer Speed = SQRT(3.28/1.28) times Wave Speed =1.600781059x Vwave
So,
Max Vsurfer = 1.600781059 x Vwave = 51.31296133 ft/sec = 34.98610999 MPH

If the ride is 400 yards = 1200 feet, the duration time of the ride = 1200/Vsurfer
So, T,ride = 1200ft / 51.31296133fps = 23.38590424 seconds.

Buzzy Trent did it in 24 seconds…I never saw anybody do it in 23 seconds.

For more Northern Latitudes like, say that of San Francisco, where g= 9.8m/s^2,
Or, g = 32.155223097 f/s^2,
Then,
Vwave = 32.07602518 ft/sec, so Max Vsurfer = 51.34669356 fps.

Then, Maximum Surfer Speed = 35.00910925 MPH.

Thus, it looks like Vmax for 25 ft waves is around 35 MPH.

Now, for a general formula for calculating Surfer Speed, given Wave Height:

Maximum Surfer Speed seems to be proportional to the SQUARE ROOT of the Breaking Wave Height, so, at the latitude of Makaha down to the South Shore (Ala Moana Bowls, and Waikiki in general), where g = 32.11 f/s^2, the Maximum Surfer Speed for a 1 foot wave scales down to about 1/5th as fast as for that of a 25 ft wave, or about 1/5th of 35 MPH. Therefore, we could use a general formula for Hawaiian waters up to Northern California surf spots, that says:

Maximum Surfer Speed, MPH = 7* SQUARE ROOT of (Hb,ft)
Or,
Maximum Surfer Speed, fps = 10.26666666*SQUARE ROOT of (Hb,ft)

Note:
The exact value of “g” required to produce the above values of 7 MPH EXACTLY for my Maximum Surfer Speed, if we use a Breaker Depth Index of exactly 1.28, is 32.1355013550…,etc. ft/sec^2, where the “13550” is a repeating decimal.

So, the exact decimal fraction is 13550/99999, and the total value of g is therefore EXACTLY given by: 99999*(32+13550/99999) = 3213518 / 99999.

The equivalent exact value of g in the metric measure is (“g” in ft/sec^2) * 0.3048
Or, g = 9.79490081301 m/s^2.
Then,
Using the “Solver” function in my TI-85 graphics calculator, and the formula for g that I described in “Part 1” of this “Surfer Speed” document, I get the following result for the Equivalent Surf Spot Latitude for my basic formula of Surfer Speed:

Exact Latitude = 32.07066048 degrees.

Only then, is Max Surfer Speed, mph = 7*SQRT(Hb,ft), EXACTLY.

So, if you want to ride a 49-ft wave, you can get up 49 MPH. The Tow-in guys are already doing that, I suppose.

But, do you want to ride a 100-foot wave? Your fastest ride might have to be up to 70 MPH. Can you still stand up? You can train by standing on top of a car on the freeway going 70 MPH. Ha!

At those high speeds, wind drag would become significant, limiting the top speed achievable by a stand-up surfer. So, it’s likely they would not be able to make it across if the Peel Angle is such that they would have to go 70mph to make it.

Now,
For all you surfers around the world who use the Metric system:

25 feet x 0.3048 = 7.62 meters

For Hb = 1 foot, the basic factor in my Surfer Speed formula is 7 MPH, and the equivalent factors used for other measurements of speed are:
7 MPH x22/15 = 10.266666666 f/s,
10.266666666 x.3048 = 3.12928 m/s,
3.12928 x3600 seconds = 11265.408 m/hour, /1000 = 11.265408 Km/hr,
11.265408 /1.852 = 6.082833693 Nautical Miles per Hour (“Knots”)

But, we want a formula in metric units that has the basic Breaking Wave Height of 1 Meter, instead of 1 foot.
I meter, in feet, is 1/0.3048 = 3.280839895 ft, so my formulas using MPH and Vfps need to be scaled up by a factor of the SQUARE ROOT of (3.280839895 / 1) = 1.81130889 times.

So, when using MPH, Vsurfer, mph = 7*SQRT(Hb,ft),
but now, Hb = 3.280839895 ft.

Then, Vsurfer,mph = 7*SQRT(3.280839895) = 12.67916223 MPH

And, since Vft/s = (22/15) times Vmph,
Then, Vft/s = (22/15)*12.67916223 MPH = 18.5961046 f/s

Since Vm/s = 0.3048 * Vft/s,
Vm/s = 0.3048*18.5961046 f/s= 5.668092683 m/s

So,
Our basic Metric Unit formula for Maximum Surfer Speed, in meters per second, is:

Vsurfer, m/s = 5.668092683 *SQRT(Hb, m)

In 1 hour, or 3600 seconds, the distance in meters travelled = 3600*Vm/s
So, Distance, m = 3600* 5.668092683 m/s = 20,405.13366 meters,
or, Distance in Kilometers, = m/1000 = 20.40513366 KM

So,
Vsurfer, km/hr = 20.40513366*SQRT(Hb,m)

The speed in Nautical Miles per Hour, or “Knots”, is Vkm/hr / 1.852
Or, Vkts = 20.40513366 / 1.852 = 11.01789075 Kts

So,
Vsurfer,kts = 11.01789075*SQRT(Hb,m)

Let’s try some examples:

For a 2-meter (true height) breaking wave, how fast could a surfer go on a fast section?
Vsurfer,km/hr = 20.40513366* SQUAREROOT (2) = 28.85721676 Km/hr

So,
For a 2-meter wave, Vsurfer, km/hr = 20.40513366*SQRT(2) = 28.85721676

For a 3-meter wave, Vsurfer, km/hr = 20.40513366*SQRT(3) = 35.34272823

For a 4-meter wave, Vsurfer, km/hr = 20.40513366*SQRT(4) = 40.81026732

This concludes Part 2

If you read all of Part 1 and Part 2, Thanks for your patience! I hope somebody tests these results with GPS devices or radar guns. It will be interesting to see just "How Fast CAN a Surfer Go?" in real life.

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